Changing Dimensions: Surface Area and Volume
Recall that similar figures have proportional side lengths. The surface areas of similar three-dimensional figures are also proportional. To see this relationship, you can compare the areas of corresponding faces of similar rectangular prisms.
Area of front of smaller prism
\[ l \cdot w \]
3 \cdot 5
15

Area of front of larger prism
\[ l \cdot w \]
6 \cdot 10
\[(3 \cdot 2) \cdot (5 \cdot 2)\]
\[(3 \cdot 5) \cdot (2 \cdot 2)\]
15 \cdot 2^2

Each dimension has a scale factor of 2.

Remember!
A scale factor is a number that every dimension of a figure is multiplied by to make a similar figure.
The area of the front face of the larger prism is $2^2$ times the area of the front face of the smaller prism. This is true for the entire surface area of the prisms.

**SURFACE AREA OF SIMILAR FIGURES**

If three-dimensional figure $B$ is similar to figure $A$ by a scale factor, then the surface area of $B$ is equal to the surface area of $A$ times the square of the scale factor.

$$\text{surface area of figure } B = \frac{\text{surface area of figure } A \cdot (\text{scale factor})^2}{}$$
Additional Example 1: Finding the Surface Area of a Similar Figure

The surface area of a box is 35 in\(^2\). What is the surface area of a similar box that is larger by a scale factor of 7?

\[ S = 35 \cdot 7^2 \quad \text{Multiply by the square of the scale factor.} \]

\[ S = 35 \cdot 49 \quad \text{Evaluate the power.} \]

\[ S = 1,715 \quad \text{Multiply.} \]

The surface area of the larger box is 1,715 in\(^2\).
Check It Out: Example 1A

The surface area of a box is 1,800 in$^2$. Find the surface area of a similarly shaped box that is smaller by a scale factor of $\text{ }.}$
Check It Out: Example 1A

The surface area of a box is 1,800 in². Find the surface area of a similarly shaped box that is smaller by a scale factor of .
The volumes of similar three-dimensional figures are also related.

The volume of the larger tank is $2^3$ times the volume of the smaller tank.
If three-dimensional figure $B$ is similar to figure $A$ by a scale factor, then the volume of $B$ is equal to the volume of $A$ times the cube of the scale factor.

$$\text{volume of figure } B = \text{volume of figure } A \cdot (\text{scale factor})^3$$
Additional Example 2: Finding Volume Using Similar Figures

The volume of a child’s swimming pool is 28 ft\(^3\). What is the volume of a similar pool that is larger by a scale factor of 4?

\[
V = 28 \cdot 4^3
\]

Multiply by the cube of the scale factor.

\[
V = 28 \cdot 64
\]

Evaluate the power.

\[
V = 1,792 \text{ ft}^3
\]

Multiply.

Estimate

\[
V \approx 30 \cdot 60
\]

Round the measurements.

\[
= 1,800
\]

The answer is reasonable.
Check It Out: Example 2

The volume of a small hot tub is $48 \text{ ft}^3$. What is the volume of a similar hot tub that is larger by a scale factor of 2?
The sink in Kevin’s workshop measures 16 in. by 15 in. by 6 in. Another sink with a similar shape is larger by a scale factor of 2. There are 231 in³ in 1 gallon. Estimate how many more gallons the larger sink holds.
Additional Example 3 Continued

Understand the Problem

Rewrite the question as a statement.

• Compare the capacities of two similar sinks, and estimate how much more water the larger sink holds.

List the **important information:**

• The smaller sink is 16 in. x 15 in. x 6 in.

• The large sink is similar to the small sink by a scale factor of 2.

• \(231 \text{ in}^3 = 1 \text{ gal}\)
Make a Plan

You can write an equation that relates the volume of the large sink to the volume of the small sink. Then convert cubic inches to gallons to compare the capacities of the sinks.

Volume of large sink = Volume of small sink \cdot (a\ scale\ factor)^3
Additional Example 3 Continued

Solve

Volume of small sink = 16 \times 15 \times 6 = 1,440 \text{ in}^3

Volume of large sink = 1,440 \times 2^3 = 11,520 \text{ in}^3

Convert each volume into gallons:

1,440 \text{ in}^3 \times \frac{1 \text{ gal}}{231 \text{ in}^3} \approx 6 \text{ gallons}

11,520 \text{ in}^3 \times \frac{1 \text{ gal}}{231 \text{ in}^3} \approx 50 \text{ gallons}

Subtract the capacities: 50 \text{ gal} - 6 \text{ gal} = 44 \text{ gal}

The large sink holds about 44 gallons more than the small sink.
Double the dimensions of the small sink and find the volume:
$32 \times 30 \times 12 = 11,520 \text{ in}^3$. Subtract the volumes of the two sinks:
$11,520 - 1,440 = 10,080 \text{ in}^3$. Convert this measurement to gallons:
$10,080 \times \frac{1 \text{ gal}}{231 \text{ in}^3} \approx 44 \text{ gal}$
Check It Out: Example 3

The bath tub in Ravina’s house measures 46 in. by 36 in. by 24 in. Another bath tub with a similar shape is smaller by a scale factor of \( \frac{1}{2} \). There are 231 in\(^3\) in 1 gallon. Estimate how many more gallons the larger bath tub holds.